

Systems Modelling of Steering System using OpenModelica

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ABSTRACT

Steering system is one of the key vehicle-driver interface that impacts driving fatigue and perception of quality by driver. It is also a safety critical system governed by several Homologation requirements. While Homologation rules mandates the steering effort to be within certain limits, for a driver, asymmetric behavior in terms of angle & torque between LH & RH turn is also undesirable. The sub-attributes are affected indirectly by packaging constraints in each vehicle platform and model. For meeting the stringent targets in each variant and to maintain minimal unique parts, several design iterations would be needed. Physical testing involves

higher cost and time. Hence development of a steering system simulation model becomes essential. This paper details the development of a steering model to simulate and analyze the steering system performance of commercial vehicles. A single steer model with hydraulic assistance is developed using OpenModelica (OM), an open source system modelling software. The parameters such as steering wheel effort, wheel lock angles, Ackermann error and Turning circle diameter (TCD) are computed along with prediction of dry park effort. Based on iterations, an optimized configuration with reduced asymmetry can be derived.

KEYWORDS: Simulation, steering linkage optimization, linkage asymmetry, static steering, steering performance prediction, friction modelling, hydraulic assistance modelling, steering friction, tire-road torque

Introduction

Steering system, in a vehicle, aids in directional stability and maneuverability. Today, most commercial vehicles come with hydraulic power assisted steering system. The system involves gear reduction for torque multiplication and mechanical linkages for transferring the motion from hand wheel to road wheel. Based on requirement, a hydraulic pump supplies pressurized fluid to the steering gearbox which acts on the surface of the piston present in it. This hydraulic assistance greatly reduces the overall steering effort to be input by the driver at the steering wheel

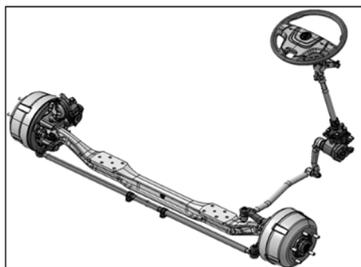


Fig. 1. Steering system with linkages

The mechanical linkages include steering column, drop arm, draglink, steering lever, track rod lever and track rod. These components form 2 trapezoidal linkages. Droparm, draglink and steering lever constitute the 1st trapezoid while track rod along with the 2 track rod levers forms the 2nd trapezoid. In order to negotiate a turn, we know that the inner wheels are required to turn by a larger angle than the outer wheel. This is achieved by the design of 2nd trapezoid based on Ackermann principle which requires the track rod levers to be inclined pointing towards the center of rear axle. (True Ackermann). While this 2nd trapezoid solves the purpose of achieving differential wheel lock angles on either side, it creates a

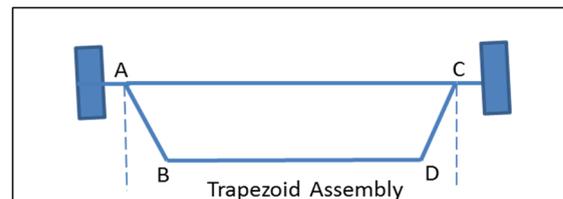


Fig. 2. Trapezium linkage at SAP

ABBREVIATIONS: TCD - Turning circle Diameter; PAS - Power Assistance; OM – OpenModelica; KPI - Kingpin Inclination; FMI - Functional Mockup Interface; KPC - Kingpin centre; RHD - Right hand drive; DA – Droparm; SAP - Straight Ahead position; FMU - Functional Mockup Unit

torque asymmetry between LH and RH turn at the steering lever. This torque asymmetry is to be compensated by suitable design of the 1st trapezoid linkages.

The above image represents the 2nd trapezoidal linkage at the track rod assembly. AB & CD represents the track rod arms while AC is the front axle and BD is the track rod. For RHD vehicles, the RH wheel is directly rotated by the steering lever. But the LH wheel is rotated through this linkage which has variable linkage ratio for LH & RH turn.

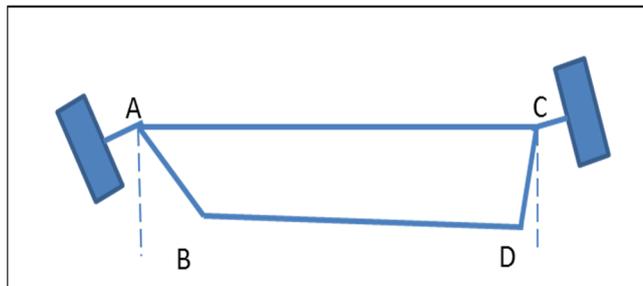


Fig. 3. Trapezium linkage during LH turn

During LH turn, the LH track rod arm (AB) moves away from the perpendicular (decreasing effective arm length), hence higher force (force = torque / decreasing radius) gets transmitted to track rod. At the same time, RH track rod arm (CD) moves towards the perpendicular. Hence the higher force gets multiplied by the increasing effective arm length (torque = force X increasing radius) resulting in higher torque requirement for the LH turn. The converse happens during an RH turn and lesser torque requirement is needed to rotate the LH wheel in an RH turn due to higher linkage ratio.

To compensate this torque asymmetry, the initial position of droparm or/ and steering lever (1st trapezoid) is altered so that we achieve symmetric torque at the steering wheel. By several iterations optimized position of drop arm/ steering lever with reduced asymmetry can be derived. In addition to torque asymmetry, packaging constraints also play a role in finalizing this optimized position.^[4] (Durstine)

In order to meet the above-mentioned requirements, a system model of steering system is built using OpenModelica software. OpenModelica is a free and open source environment based on the object-oriented Modelica language for modeling, simulating, optimizing and analyzing complex dynamic systems. It is very easy to create very large parametrized models using component arrays of models. It is based on system equations and states. Equations defined in Modelica are acausal (direction is not specific) in nature which reduces the complexity in defining them. Thus, one can describe technical systems realistically with equations based on physical principles. OpenModelica tool provides a number of facilities such as debugging, optimization, visualization and 3D animation. It also supports FMI which allows import/export of models from/to other system modelling software.

Vehicle Specifications

TABLE 1
Vehicle and steering system specification

| Parameters | Description | Units |
|------------------------------|-----------------------------------|-------|
| Vehicle Model | 6X2 Haulage | - |
| Wheel Base | 6300 | mm |
| Wheel track - Front | 2080 | mm |
| GVW – Laden | 28000 | kg |
| Tyre spec. Front/Rear | 10R20 | - |
| Type of steering system | Hydraulic power assisted steering | - |
| Type of steering column type | Tilttable & Telescopic | - |

Model Overview

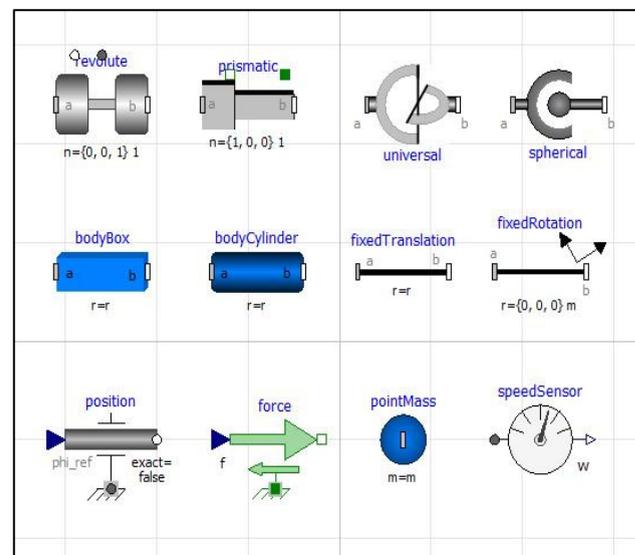
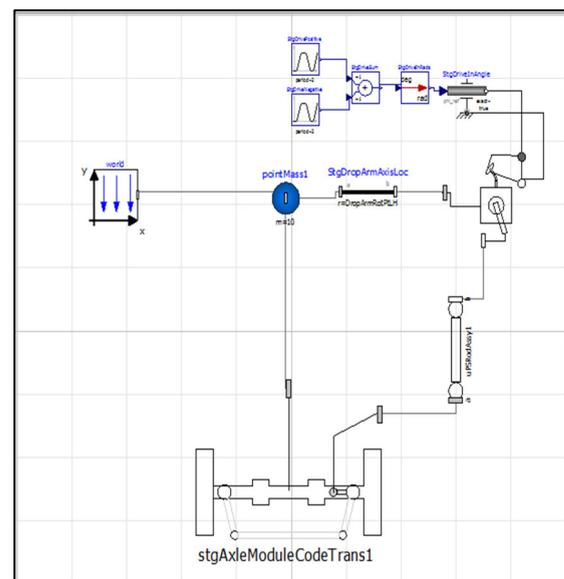


Fig. 4. Model overview & library components used

Assumptions/Deviations

Following assumptions/deviations are considered in the model:

- Axle’s position is fixed. Wheel has the freedom to move. So jacking effect occurs as vertical downward displacement of wheel while axle does not get lifted up as in reality
- Total system friction is given at kingpin based on test data
- Tire-road friction torque is also arrived based on test data and is given at tire centre.
- Hydraulic assistance is considered at 80% efficiency.

Model Building

The modelling of steering system involves the following 4 steps.

- Linkage modelling
- Hydraulic assistance modelling
- Linkage friction modelling
- Tire road friction modelling

Linkage Modelling: The different components of the steering system are built separately and then assembled together finally to arrive at the complete steering system starting from steering wheel to road wheel.

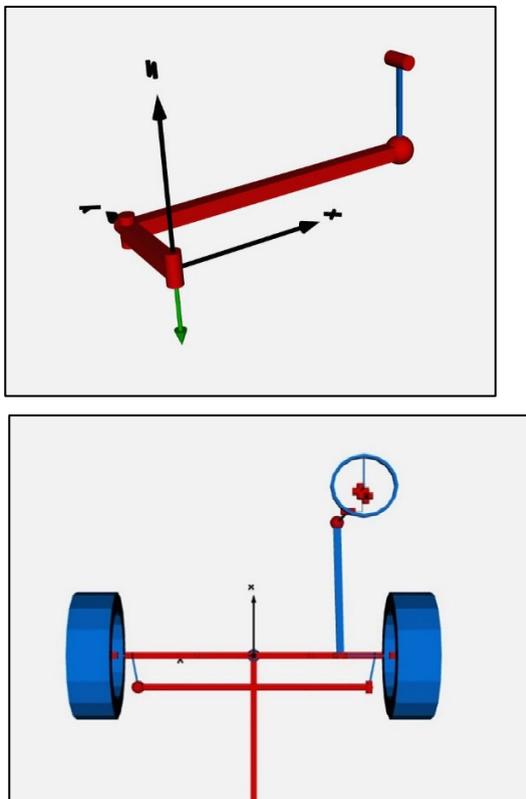


Fig. 5. Droparm-draglink-steering lever and Track rod linkage assembly

“A sinusoidal steering input is given at the steering wheel. The angular input is transferred to the steering gearbox through the steering column.” Power assistance acts based on the input hand wheel torque. The drop arm motion is converted to linear motion by the draglink which then rotates the steering lever about the kingpin. The right-hand wheel is turned by the axle arm while motion to the LH wheel is transferred to the track rod arms and track rod lever assembly.

The dimensions and specifications of all components need to be input to the model. The front axle load, steering system friction, tire-road friction and other vehicle specifications are also to be provided. Steering geometry such as Caster, KPI, Camber can also be varied. On running the simulation, a 3D visualization of the system is generated and output parameters such as Hand wheel torque, wheel lock angles, Ackermann error and TCD are computed.

By studying the different output parameters, modifications can be done and the results of simulation can be visualized.

Hydraulic assistance modelling: The steering pump delivers pressurized fluid to the steering gearbox which provides assistance to manual steering. The amount of assistance offered is defined by the power assistance curve. This curve plots the “delivery pressure” against the “input hand wheel torque”. The power assistance curve is given by the suppliers and it is based on the torsion bar and valve assembly inside the steering gearbox. The delivery pressure for a given amount of handwheel torque is defined by this curve. Based on this pressure, the amount of hydraulic assistance is varied.

From power assistance curve measured in physical testing, “Handwheel torque vs Pressure” is obtained. Pressure is converted into assistance torque considering the max output torque of the gearbox. Thus “Handwheel torque vs Assistance torque” is obtained for LH & RH turn which is input to the steering gearbox of the model.

Through simulation, this curve can be modified and studied to get the desired output parameters of steering performance. Once the assistance curve is finalized, feasibility can be checked with the suppliers to get the closest actual curve possible to the desired one.

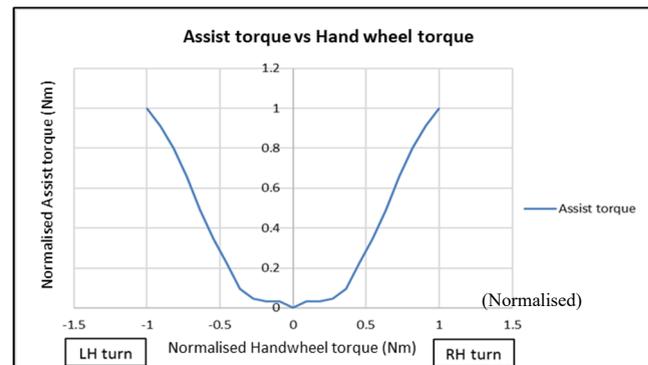


Fig. 6. Assistance torque vs Hand wheel torque

Forces Involved in a Steering System

There are 3 primary forces that resist the steering action.

- (i) Jacking force – As we steer the vehicle, the tires rotate about the kingpin which is inclined about the x and y axis. Rotation about this inclined surface pushes the wheel vertically downwards. Once the maximum tire deflection is reached, the additional downward force cannot push the wheel further down. Hence the reaction force causes the vertical upward movement of the axle relative to the ground. This creates an effect of jacking up the vehicle and is termed as “jacking force”.
- (ii) Linkage friction – Linkage friction includes the steering gear friction, kingpin friction and ball joint friction that act during steering a vehicle.
- (iii) Tire-road friction – The friction between the tire and the road surface is a major resisting force. It depends on the load acting on tire, tire properties and the road surface.

Linkage friction modelling: Linkage friction is calculated from Air bearing test data and is input to the model.

Air bearing is a friction-less surface. The tires are placed on air bellows that prevent tire-road contact and offer a zero-friction surface for the tires. This eliminates the tire road friction while maintaining the same ground clearance unlike jacking the wheels. In this test condition, jacking force and linkage friction are the only forces that resist the steering action. The total system friction is provided at the kingpin in the model.

During LH or RH turn (SAP to max lock position),

$$\text{Total required torque during turn} = \text{Jacking torque} + \text{Friction torque} \dots(1)$$

During return (Return from max lock to SAP position),

$$\text{Total required torque during return} = \text{Jacking torque} - \text{Friction torque} \dots(2)$$

Thus (1-2),

$$\text{Friction torque} = (\text{Total required torque during turn} - \text{Total required torque during return})/2$$

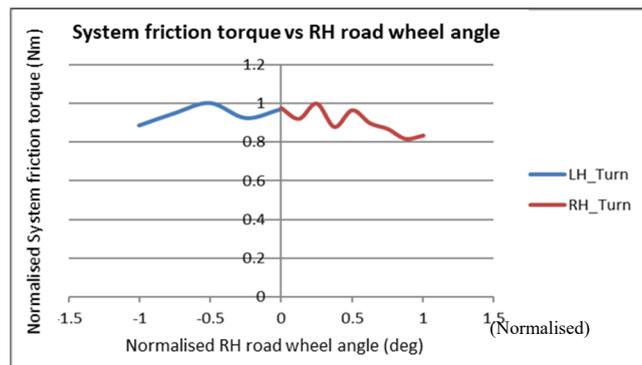


Fig. 7. System friction torque

Tire – road friction modelling: In addition to the system friction, the friction between tire and road also resists a turn. This torque is also computed using Air bearing and Static parking test data.

Static parking test is a test where the vehicle is steered in a static condition to measure the torque requirement for parking maneuvers and to move out from a parking bay. In this condition, tire-road friction torque acts in addition to jacking and system friction.

Thus, by comparing results of Air bearing and Static parking test, the tire road friction is estimated and is input at the tire centre in the model

$$\begin{aligned} \text{Torque at droparm(DA) to overcome Tire-road} \\ \text{friction torque} &= \text{Torque at DA during Dry park test} \\ &- \text{Torque at DA during Air bearing test} \\ \{ \text{Torque at droparm} &= (\text{Hand wheel torque} * \\ \text{Steering gearbox ratio}) &+ \text{Hydraulic assistance} \\ \text{torque} \} \end{aligned}$$

Hydraulic assistance torque is calculated from steering system pressure considering direct proportionality. (Ex. If Steering gearbox max torque specification is 3000Nm @ 120 bar pressure, the assistance torque at 80 bar would be (3000/120)*(80) = 2000Nm)

From this torque at droparm, we can calculate the actual tire road friction torque using the linkage ratio of the steering components.

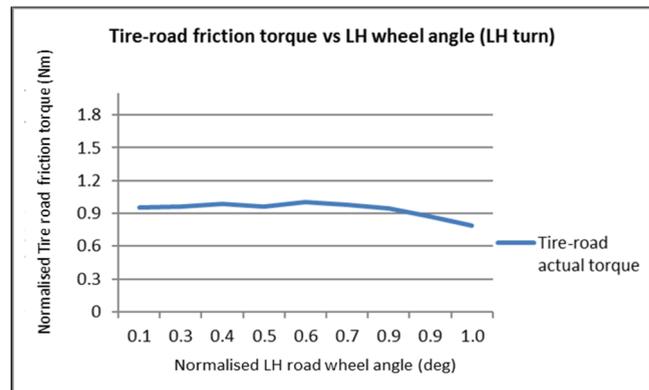


Fig. 8. Tire road friction torque – LH turn

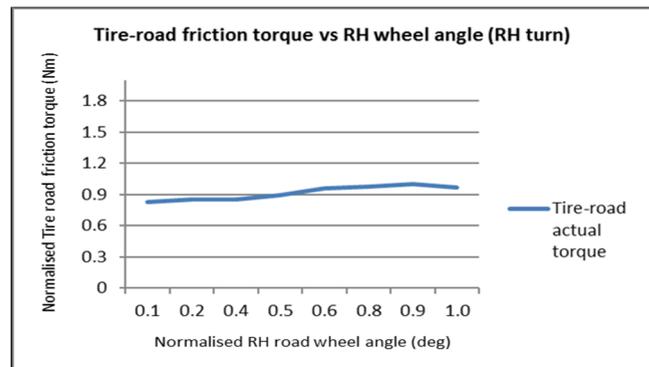


Fig. 9. Tire road friction torque – RH turn

Model Verification

In order to verify the correctness of model, the results of simulation were checked with 3D CAD software kinematics output. In addition to this the effect of several parameters were studied to ensure if the results match the expected theoretical trend thus affirming the model's correctness. The following verification steps were done:

- Verification of kinematic linkage
- Effect of Kingpin inclination (KPI)
- Scrub radius and its effect

Verification of kinematic linkages: The output – wheel angles against the steering wheel input was compared with 3D CAD software results. The results of the model matched closely to the values obtained by kinematic analysis in the 3D CAD software model. Correlation was greater than 90% and hence kinematic correctness of the model was verified.

Effect of Kingpin inclination (KPI): Kingpin is the component about which the stub axle (wheel end) rotates. Generally, the kingpin is inclined about X and Y axes. The inclination about X axis when viewed from the vehicle front with respect to the vertical is called as King pin inclination. KPI helps in returnability of the steering.

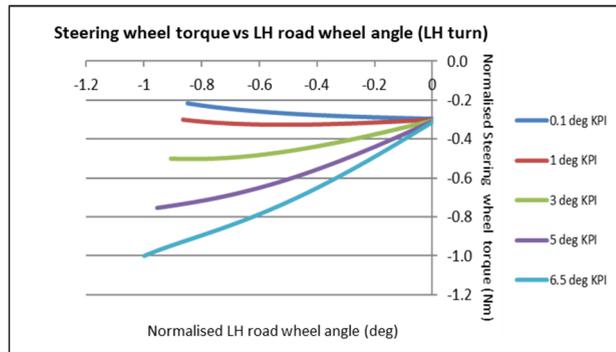


Fig. 10. Effect of KPI – LH turn

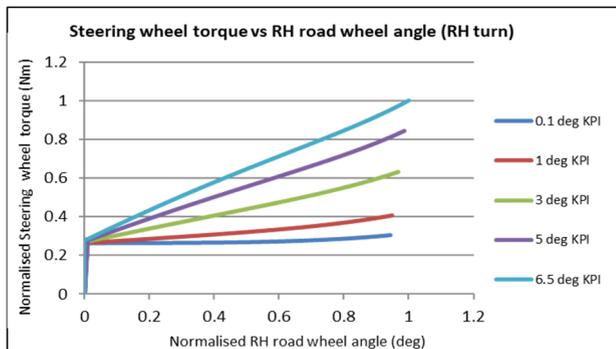


Fig. 11. Effect of KPI – RH turn

It can be seen that with increase in KPI angle, the steering wheel torque is progressively increasing as expected.

Due to the kingpin inclination, there occurs a vertical displacement(z) as the wheel turns. This is called as jacking. Vertical displacement of wheel with turn is

observed in the model and is also found to increase with increasing KPI as expected.

Increasing in steering wheel torque and vertical displacement of wheel with increasing KPI affirms the modelling of kingpin and wheel end.

Scrub radius and its effect: Scrub radius is the distance between tire centre and the point where kingpin axis meets when projected to the ground. The movement of tire centre (in x & y) during a turn is studied for different KPI. Using the wheel trace points, the scrub radius of simulation is found and compared against the calculated scrub radius. The change in scrub radius with change in KPI (for fixed “KPC to tire center” length) is tabulated below.

TABLE 2
Scrub radius – Theoretical vs Simulation

| KPI (deg) | Simulation – scrub radius (m) | | | | Calculated scrub radius (m) |
|-----------|-------------------------------|----------|----------|----------|-----------------------------|
| | LH turn | | RH turn | | |
| | LH wheel | RH wheel | LH wheel | RH wheel | |
| 6.5 | 0.094 | 0.0944 | 0.0944 | 0.094 | 0.08 |
| 5 | 0.1016 | 0.102 | 0.1019 | 0.1017 | 0.092 |
| 3 | 0.1138 | 0.114 | 0.1139 | 0.1137 | 0.1107 |
| 1.5 | 0.1246 | 0.125 | 0.1245 | 0.1246 | 0.1239 |

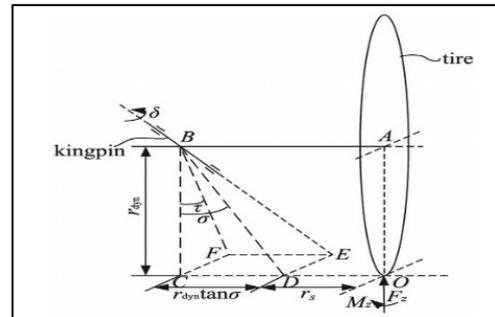


Fig. 12. Scrub radius calculation^[1] (Biao Ma, 2016)

$$\text{Scrub radius} = AB - r_{dyn} \cdot \tan \sigma$$

Reduction of scrub radius with increase in KPI and closeness to the theoretically calculated value further affirms the correctness of kingpin and wheel end.

Validation/Correlation

Once the correctness of the model was verified with the above-mentioned theoretical study, the results were validated with test data of a single steer vehicle. Validation was done in a sequential manner starting with power off condition on an air bearing which doesn't involve hydraulic assistance and tire road friction input. On achieving correlation, the power on condition on air bearing and dry park test which includes the tire road friction input were validated.

- Air bearing - power off test
- Air bearing - power on test
- Dry park - power on test

Vehicle input parameters

The following parameters were input to the developed steering model:

- Vehicle model
- Wheel base
- Wheel track – Front
- FAW – Laden
- Tyre type
- Steering gearbox ratio
- Steering wheel diameter
- Kinpin to kingpin distance
- Droparm length
- Draglink length
- Steering arm length
- Axle arm length
- Trackrod arm length
- Trackrod length
- KPI
- Caster
- System friction torque
- Tire-road friction torque (Road surface – concrete)
- PAS curve

Air bearing - power off test

Test procedure: Vehicle front wheels are parked over the air bearing which is a frictionless surface. Adequate air supply is given to air bearing to lift the vehicle. The vehicle freely floats over the air bearing surface which removes the tire-road interaction and frictional resistance offered by road surface. The force required to turn the steered wheels in this condition from lock to lock position is measured at the steering end. No linkages shall be disturbed. The test is conducted in Engine off condition with a consistent rate of steering input.

The steering wheel torque of test vs simulation results are plotted for LH & RH turn in the above graphs. Since simulation is an ideal condition, the mechanical efficiency

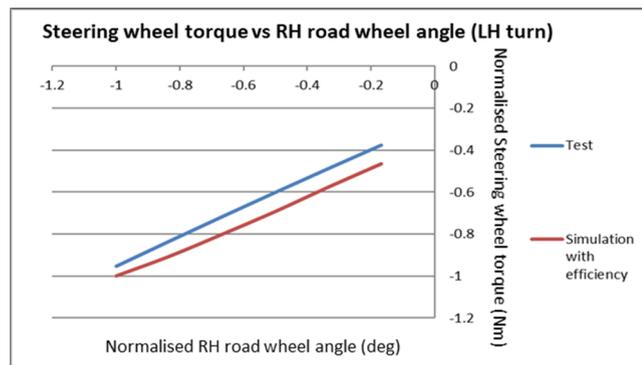


Fig. 13. Air bearing PAS off – Test vs Simulation – LH turn

of the gearbox and linkages are included. It can be found that the results are closer at higher wheel angles compared to very low angles. Correlation of 90% is achieved between physical test and simulation results.

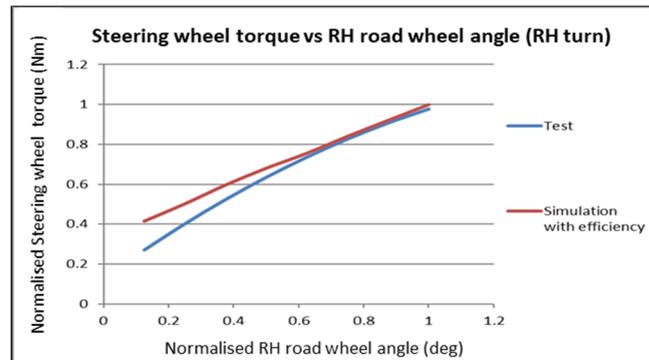


Fig. 14. Air bearing PAS off – Test vs Simulation – RH turn

Air bearing - power-on test

Test procedure: Vehicle front wheels are parked over the air bearing which is a frictionless surface. Adequate air supply is given to air bearing to lift the vehicle. The vehicle freely floats over the air bearing surface which removes the tire-road interaction and frictional resistance offered by road surface. The force required to turn the steered wheels in this condition from lock to lock position is measured at the steering end. No linkages shall be disturbed. The test is conducted in Engine on condition with a consistent rate of steering input.

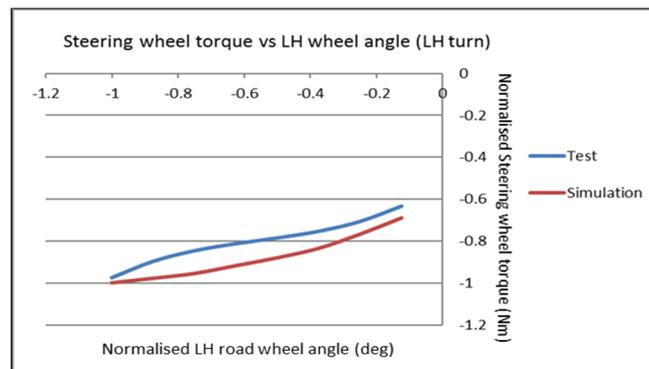


Fig. 15. Air bearing PAS on – Test vs Simulation – LH turn

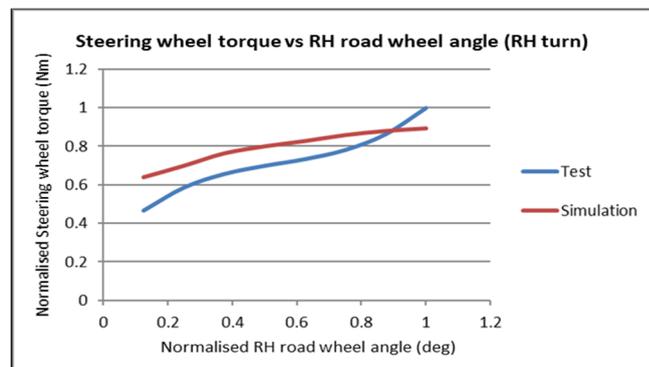


Fig. 16. Air bearing PAS on – Test vs Simulation – RH turn

The steering wheel torque of test vs simulation results of power on air bearing test condition are plotted for LH & RH turn in the above graphs. Correlation of 80-90% is achieved between physical test and simulation results.

Dry park - power on test

Test procedure: Static steering effort is measured by turning the steering wheel from lock to lock with vehicle parking brake applied. Maximum effort value before the peak torque is measured.

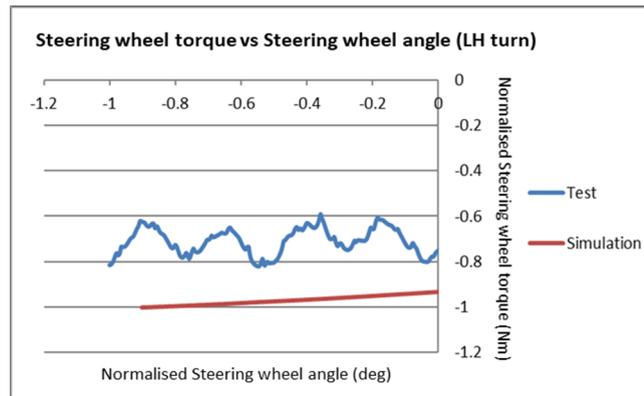


Fig. 17. Dry Park PAS on – Test vs Simulation – LH turn

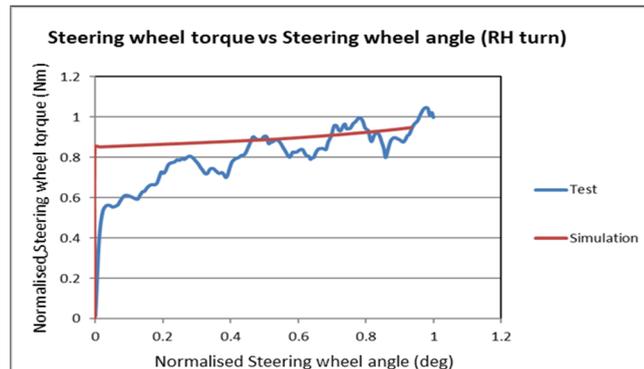


Fig. 18. Dry Park PAS on – Test vs Simulation – RH turn

The steering wheel torque of test vs simulation results of power on static parking test condition are plotted in the above graphs. The test and simulation results match closely for RH turn compared to LH turn.

Output Parameters

The wheel lock angles, turning circle diameter and Ackermann error values computed by simulation were validated with the test results and found to have correlation of over 80%.

The road wheel angles are plotted against the input steering wheel angle. The steering input required and the corresponding road wheel angles for LH & RH turn can be computed for necessary study and improvements.

The CMVR regulations mandate TCD to be less than 24m for Heavy vehicles. In simulation, the turning circle diameter is computed from the road wheel angles and using the vehicle wheel base and wheel track inputs. The computed TCD is tabulated against the TCD measured during physical tests.

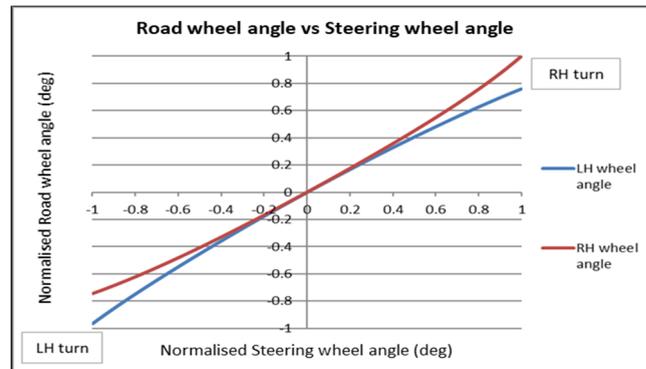


Fig. 19. Steering wheel angle vs Road wheel angle

TABLE 3

TCD – Test vs Simulation

| Parameter | LH turn | | RH turn | |
|-----------|---------|------------|---------|------------|
| | Test | Simulation | Test | Simulation |
| TCD (m) | 23.5 | 23 | 23.7 | 23.2 |

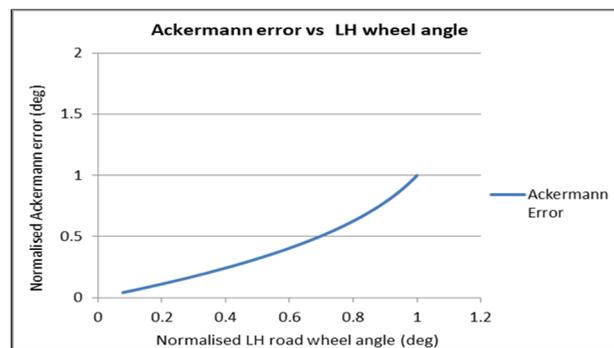


Fig. 20. Ackermann error – LH turn

Ackermann error is plotted against the road wheel angle for LH & RH turn.

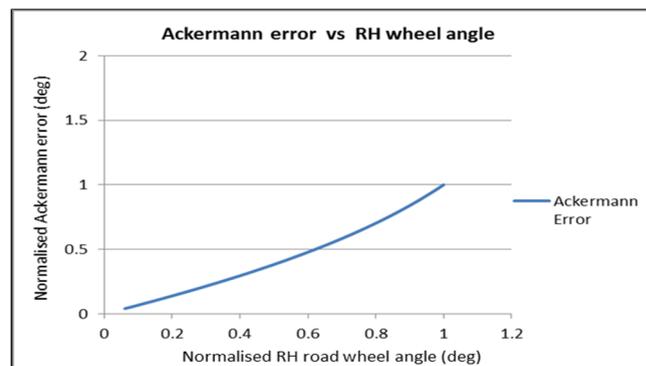


Fig. 21. Ackermann error – RH turn

Benefits of the Model

Study of complex steering geometry

Parametric modelling helps in performing quick iterations just by varying the magnitude of required variables. KPI, Caster, Camber, Scrub radius are the key steering geometry. Variation of one parameter has an

effect on the others and the entire steering performance. For example, increasing the KPI will reduce the scrub radius for a given axle arm and wheel offset. Increase of KPI, increases torque as seen in previous study. But reduction of scrub radius lowers the torque requirement. The net effect on steering system is of our concern. Study of these phenomenon through physical testing requires front axle machining which might take up higher time and cost Hence the model serves to address such complex real-life problems and solutions.

Gearbox output torque requirement prediction

Gearbox output torque is the key parameter based on which a gearbox is chosen for a particular application. Load requirement will vary based on the application. Hence it is critical to choose the right gearbox for the right application.

The torque output at drop arm which is the sum of hydraulic assistance and mechanical torque (handwheel torque * steering ratio) should be sufficient to resist the different loads acting against the steering system. By knowing the torque requirement at droparm, the gearbox with sufficient output torque can be chosen using this model.

In power off condition, the simulation is run with system friction and tire road friction torque with required front axle load. This is similar to performing static park test in power off condition. In this condition, the torque at droparm output is studied for LH & RH turn. This indicates the torque requirement for the given load conditions. Based on this the right gearbox (with sufficient max output torque) for the application can be chosen.

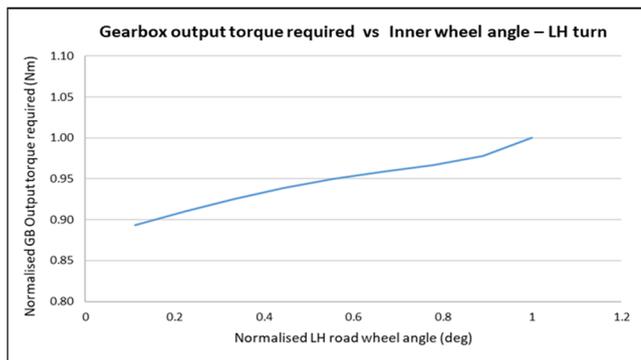


Fig. 22. Gearbox output torque requirement – LH turn

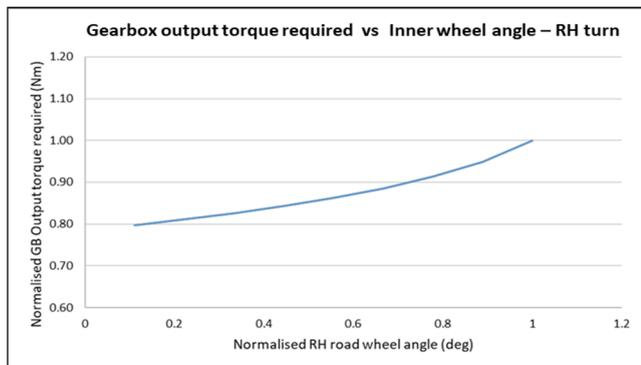


Fig. 23. Gearbox output torque requirement – RH turn

Droparm & steering lever position optimization

Droparm and steering lever position at SAP is a critical parameter with respect to torque asymmetry. As seen earlier, the track rod assembly has an inherent asymmetry. For RHD vehicles, the torque requirement is higher during LH turn compared to RH turn due to this trapezoidal assembly.

To compensate this, initial position of droparm or/and steering lever is varied such that it reduces the LH turn torque and increases RH turn torque requirement. At the same time the number of steering wheel turns to achieve maximum wheel lock in LH & RH turn should also be fairly close enough. i.e while addressing torque asymmetry, we should also have in mind the the steering wheel angle asymmetry. There are also other constraints like packaging and angle formed by the linkages at wheel lock.

It is a trade-off between torque and angle. But by iterations we can arrive at positions where we can achieve both angle and torque symmetry.

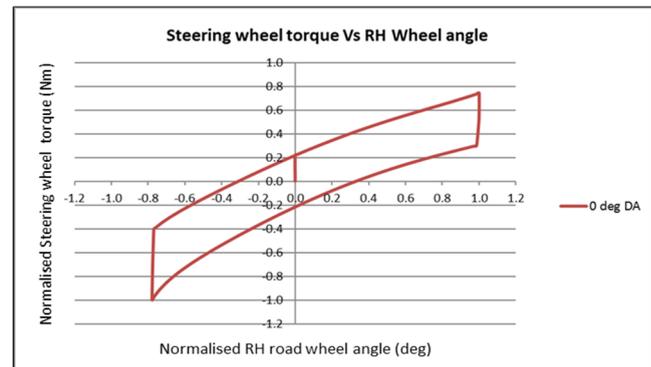


Fig. 24. Steering wheel torque vs RH wheel angle – Droparm at 0 deg position

The graph represents the steering wheel torque for equal inner wheel lock angle on either sides for drop arm position at 0 degree (perpendicular to ground) during SAP

The above graph shows the inherent asymmetry of the steering system. With drop arm at 0 degree, it is symmetric for LH & RH turn. The asymmetry in the track rod assembly (higher LH turn torque) is reflected directly at the steering wheel as drop arm is perfectly symmetric. By varying the position of droparm or/and steering lever suitably, we can achieve symmetric torque for LH & RH turns.

Torque symmetry: By Droparm position variation

By rotating the droparm initial position towards Front axle, we are able to achieve lower torque at LH turn. This droparm orientation compensates the higher LH turn torque at the steering lever end.

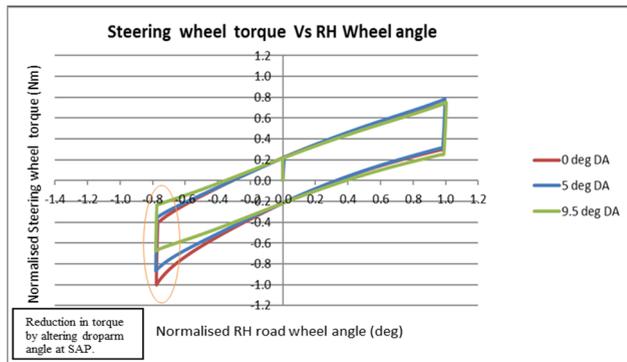


Fig. 25. Torque symmetry by droparm position variation

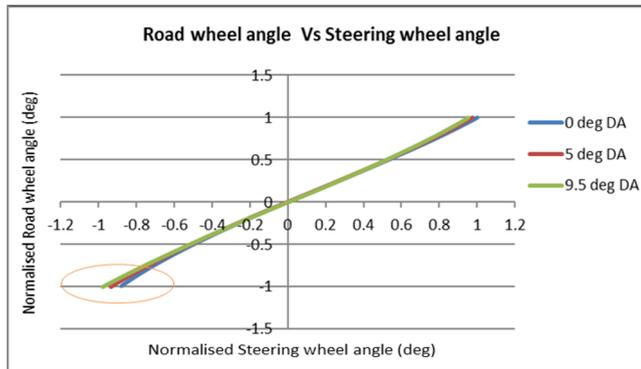


Fig. 26. Angle symmetry by droparm position variation

While rotating the droparm towards front axle reduces torque requirement for LH turn, it also increases the number of steering wheel turns to achieve the same LH wheel lock. From the graph we can say, 9.5 deg DA position requires the highest steering wheel degree to achieve the constant LH wheel lock angle. While 0 deg DA requires lesser steering wheel rotation to reach the same LH wheel lock angle.

Torque symmetry: By Steering lever position variation

To compensate the inherent asymmetry, the initial position of the steering lever can also be modified instead of the drop arm. In this case, rotating the steering lever towards vehicle front reduces the LH turn torque requirement.

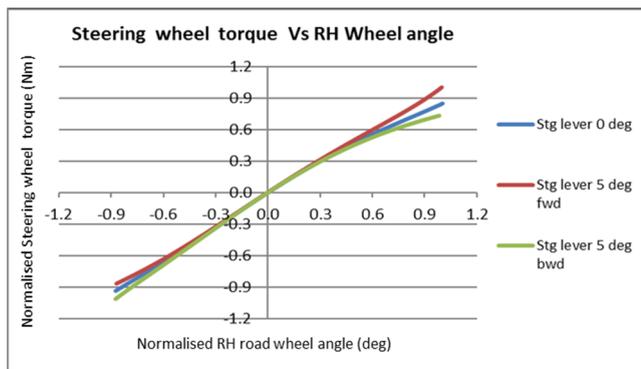


Fig. 27. Torque symmetry by steering lever position variation

In the spec of 0 deg, LH turn torque is found to be higher. With steering lever at 5 deg towards front, LH turn torque reduces while RH turn torque increases from base spec. In the same way, with steering lever at 5 deg towards rear, LH turn torque further increases and RH turn torque reduces from base spec.

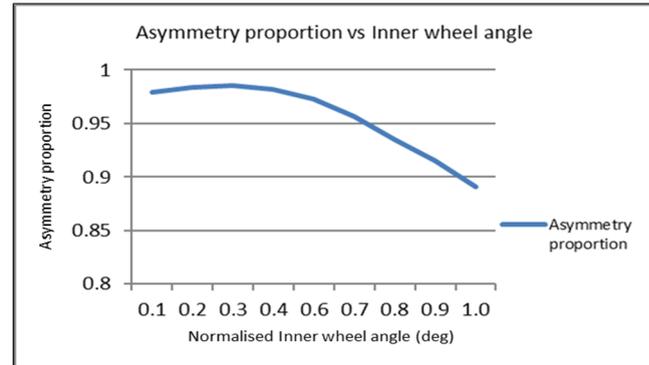


Fig. 28. Torque asymmetry proportion - Droparm at 9.5 deg position

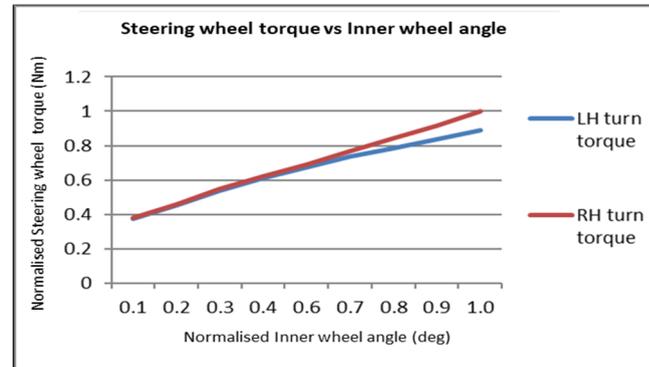


Fig. 29. Steering wheel torque vs Inner wheel angle - Droparm at 9.5 deg position

By modifying the initial position of drop arm at SAP to 7.5 deg towards front axle, better torque symmetry is achieved throughout the range of wheel angle.

With this initial position of droparm, torque asymmetry is close to unity till the wheel lock angle.

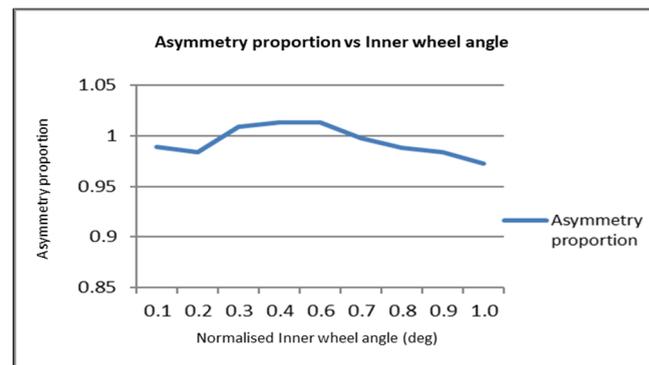


Fig. 30. Torque asymmetry proportion - Droparm at 7.5 deg position

With this drop arm position, steering wheel turn is also symmetric between LH & RH turn to achieve the same wheel lock angle.

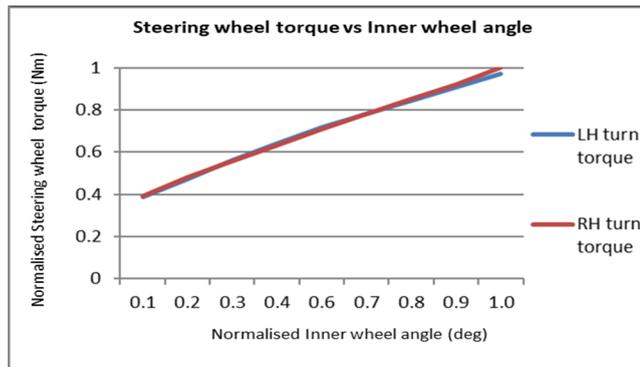


Fig. 31. Steering wheel torque vs Inner wheel angle - Droparm at 7.5 deg position

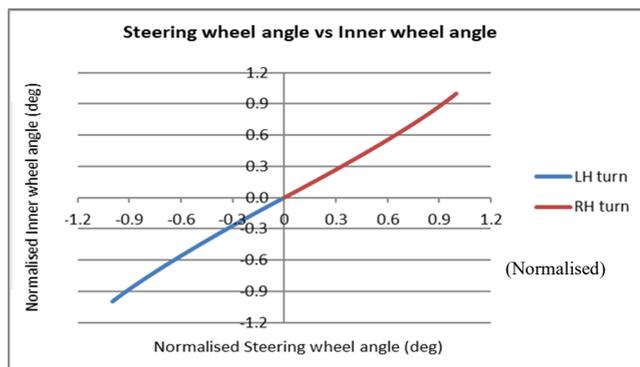


Fig. 32. Steering wheel angle vs Inner wheel angle - Droparm at 7.5 deg position

Summary/Conclusions

The usage of 2D analytical calculations for steering performance prediction were not sufficient and did not correlate to the physical tests at the expected level. Thus, 3D modelling and computation of steering performance parameters was critical. System modelling was chosen as the main advantage is that it is completely parametric unlike 3D CAD software which require physical alteration of the components to simulate various iterations. OpenModelica also had the advantage of being an open-source software and gave us the opportunity to experiment based on our needs. The developed steering model gives all outputs like Ackermann error, asymmetry and torque demand immediately and can be iterated with multiple configurations just by input of the required variables.

The model can be used for static effort and turning circle diameter prediction. Correlation of 80-90% is achieved between physical test and simulation results in static effort measurement trials.

From development perspective, we can study the variations in steering performance and finalize on the critical steering geometry. The finalization of drop arm and steering lever position at SAP to achieve torque and angle symmetry can be arrived with minimum time

and cost as opposed to physical testing. The model also helps in finalizing the gearbox capacity required for a particular application and also in determining the desirable power assistance curve. Thus, it provides a testing environment where several iterations can be performed to choose the best feasible solution.

Further Scope

The single steer model has been extended to simulate twin steer vehicles as well. The tire model can be improved to detail the tire-road interaction or custom tire models available can be included. The steering gearbox can also be replaced with FMU provided by suppliers for co-development at design stage. The study of power assistance curve and steps to improve the accuracy in dry park test conditions are to be carried out.

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